Multi-scattering effects in (e,e′p) knockout

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Abstract

Multistep direct scattering of the ejectile proton is considered in (e,e′p) reactions in the quasi-free region as an improvement over the usual treatment of final-state interactions by means of an optical potential. The theory is applied to ^{40}Ca(e,e′p) as a case example. Important contributions of two- and three-step processes are found at high missing energy and momentum in agreement with the experimental trend.

PACS numbers: 25.30.-c, 24.60.Gv

1 Introduction

Nucleon knockout by a high-energy electron in the quasi-free (QF) regime has extensively been used as a powerful tool to investigate single-particle properties of nuclei. The reason is that the electromagnetic interaction with nucleons is well known from quantum electrodynamics and, in the one-photon-exchange approximation and neglecting final-state interactions, the coincidence cross section for a detected electron of energy $E_{k'}$ and angle $\Omega_{k'}$ and detected nucleon of angle Ω has a factorised form:

$$\frac{\mathrm{d}^3\sigma}{\mathrm{d}\Omega_{k'}\mathrm{d}E_{k'}\mathrm{d}\Omega} = K\sigma_{\mathrm{ep}}S(\vec{p}_m, E_m), \qquad (1)$$

where K is a kinematical factor, $\sigma_{\rm ep}$ the elementary (off-shell) electron-nucleon cross section and $S(\vec{p}_m, E_m)$ the hole spectral function depending on the missing momentum $\vec{p}_m$ and energy E_m. The momentum $\vec{p}_m$ is also the recoil momentum of the residual nucleus and E_m its excitation energy with respect to the target nucleus. Excitation spectra and momentum distributions of the produced hole have been measured for a variety of nuclei along the whole periodic table (for a recent review, see ref. [1]).

However, to extract from the data precise information, such as e.g. the values of spectroscopic factors, an accurate treatment of final-state interactions (FSI) is necessary with the result that the simple factorisation (1) is destroyed [2]. In the one-photon-exchange approximation the general expression of the coincidence unpolarized cross section can be written in terms of four structure functions W_i as [1]

$$
\frac{{\rm d}^3\sigma}{{\rm d}\Omega_{k'}{\rm d}E_{k'}{\rm d}\Omega} = \frac{2\pi^2\alpha}{|\vec{q}|}\Gamma_{\rm V}\,K\{W_{\rm T} + \epsilon_{\rm L}\,W_{\rm L}
$$

$$
+\sqrt{\epsilon_{\rm L}(1+\epsilon)}W_{\rm TL}\cos\phi + \epsilon\,W_{\rm TT}\cos 2\phi\}, \qquad (2)
$$

where $\Gamma_{\rm V}$ is the flux of virtual photons, ϕ the out-of-plane angle of the proton with respect to the electron scattering plane,

$$
\epsilon = \left[1 + 2\frac{|\vec{q}|^2}{Q^2}\tan^2\tfrac{1}{2}\theta\right]^{-1}, \qquad \epsilon_{\rm L} = \frac{Q^2}{|\vec{q}|^2}\epsilon, \qquad (3)
$$

and $Q^2 = |\vec{q}|^2 - \omega^2$ is the negative mass squared of the virtual photon defined in terms of the momentum $\vec{q}$ and energy ω transferred by the incident electron through a scattering angle θ. In plane-wave impulse approximation, all structure functions become proportional to the hole spectral functions and eq. (1) is recovered.

It turns out that for removal of valence protons a distorted-wave impulse approximation (DWIA) is a suitable one [1, 3]. On the contrary, at high missing energy and/or high missing momentum clear evidence for a better approximation has been accumulated [4]-[7]. In addition, other processes beyond the simple one-body mechanism become important above the threshold of two-nucleon emission and in the socalled dip-region, i.e. in the region between the QF peak and the Δ-resonance excitation [8, 9].

In the dip region the semi-inclusive ^{12}C(e,e$'$p) data of ref. [9] have been compared with two calculations, one focusing on two-body meson-exchange and Δ currents [10], and the other one on short-range correlations [11]. However, in this region many-body effects leading to multi-nucleon emission are important. Limitations of the two-body process as a mechanism for understanding the (e,e$'$p) reaction have been indicated in refs. [12, 13].

In the QF region the one-body mechanism is dominant, but multiple scattering of the ejected proton with the residual nucleus is also important as shown by the large effects introduced by DWIA. A detailed analysis of the effects of multiple scattering has only been performed in a classical approach by means of a Monte Carlo study [14] where an (e,e$'$p) reaction in a given nucleus is simulated by taking into account multiple Coulomb and nuclear scattering by the outgoing proton while crossing through the residual nucleus. A quantum-mechanical treatment of FSI is proposed in the present paper taking advantage of the multistep direct (MSD) scattering theory of Feshbach, Kerman and Koonin [15].

The MSD theory has been extensively applied to describe the continuum spectrum in nuclear reactions for energies up to the pion threshold (see [16] and references therein; [17]-[24]) establishing the validity of the theory over a wide range of energies and target nuclei. The reactions are described as a series of two-body interactions between the projectile and the target nucleons leading to the excitation of intermediate states of increasing complexity. At each stage a nucleon may be emitted contributing to the pre-equilibrium energy spectrum. The theory combines a quantum-mechanical treatment of multistep scattering with statistical assumptions that lead to the convolution nature of the multistep cross sections and enables the calculation of higher order contributions – up to six steps – which would otherwise be impracticable.

In the present paper we apply the multistep scattering theory to describe the continuum spectrum of the QF (e,e$'$p) knockout reaction. Following the electromagnetic interaction between the scattered electron and the target nucleus, a target nucleon is excited to the continuum with energy E_1 and angle Ω_1 and subsequently undergoes a series of two-body interactions with the residual nucleons before being emitted with energy E and angle Ω. We aim to give a quantitative estimate of the multi-scattering effects in the high

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missing energy region as a first step in the study of FSI.

In sect. 2 the MSD theory is briefly recalled and adapted to describe the proton emission in (e,e′p) reactions. Calculational details are given in sect. 3 and the results obtained in a case example are discussed in sect. 4.

2   Theory

The MSD theory has been described in detail in refs. [16, 25] so here we give a brief account of the theoretical formalism adapted to (e,e′p) reactions without details of the derivations. The average cross section for an ejectile electron of energy $E_{k'}$ and angle $\Omega_{k'}$ and ejectile proton of energy E and angle Ω is written as an incoherent sum of a one-step and multistep (n-step) terms

$$\frac{\mathrm{d}^4\sigma}{\mathrm{d}\Omega_{k'}\mathrm{d}E_{k'}\mathrm{d}\Omega\mathrm{d}E} = \frac{\mathrm{d}^4\sigma^{(1)}}{\mathrm{d}\Omega_{k'}\mathrm{d}E_{k'}\mathrm{d}\Omega\mathrm{d}E} + \sum_{n=2}^{\infty} \frac{\mathrm{d}^4\sigma^{(n)}}{\mathrm{d}\Omega_{k'}\mathrm{d}E_{k'}\mathrm{d}\Omega\mathrm{d}E}, \tag{4}$$

where the n-step term is given by a convolution of the direct (e,e′p) knockout cross sections and one-step MSD cross sections over all intermediate energies E_1, E_2... and angles Ω_1, Ω_2... obeying energy and momentum conservation rules:

$$
\begin{aligned}
\frac{\mathrm{d}^4\sigma^{(n)}}{\mathrm{d}\Omega_{k'}\mathrm{d}E_{k'}\mathrm{d}\Omega\mathrm{d}E} &= \left(\frac{m}{4\pi^2}\right)^{n-1} \int \mathrm{d}\Omega_{n-1} \int \mathrm{d}E_{n-1} E_{n-1} \ldots \\
&\times \int \mathrm{d}\Omega_1 \int \mathrm{d}E_1 E_1 \frac{\mathrm{d}^2\sigma^{(1)}}{\mathrm{d}\Omega dE}(E, \Omega \leftarrow E_{n-1}, \Omega_{n-1}) \ldots \\
&\times \frac{\mathrm{d}^2\sigma^{(1)}}{\mathrm{d}\Omega_2 \mathrm{d}E_2}(E_2, \Omega_2 \leftarrow E_1, \Omega_1) \frac{\mathrm{d}^4\sigma}{\mathrm{d}\Omega_{k'}\mathrm{d}E_{k'}\mathrm{d}\Omega_1 dE_1}. \tag{5}
\end{aligned}
$$

The cross section for the (e,e′p) direct knockout reaction is given by eq. (2) after having included the energy distribution (see eq. (8) below). The one-step MSD cross sections for the subsequent NN scatterings are calculated by

extending the DWBA theory to the continuum and can be written as

$$\frac{\mathrm{d}^2\sigma^{(1)}}{\mathrm{d}\Omega\mathrm{d}E}(E,\Omega \leftarrow E_0,\Omega_0)$$

$$= \sum_J (2J+1)\rho_{\mathrm{1p1h},J}(U)\left\langle\frac{\mathrm{d}\sigma(E,\Omega \leftarrow E_0,\Omega_0)}{\mathrm{d}\Omega}\right\rangle_J^{\mathrm{DWBA}}, \quad (6)$$

where J is the orbital angular momentum transfer, $\langle\mathrm{d}\sigma/\mathrm{d}\Omega\rangle_J^{\mathrm{DWBA}}$ is the average of DWBA cross sections exciting 1p1h states consistent with energy, angular momentum and parity conservation and $\rho_{\mathrm{1p1h},J}(U)$ is the density of such states with residual nucleus energy $U = E_0 - E$. The latter is factorised into a level-dependent density and a spin distribution, $\rho_{\mathrm{1p1h},J}(U) = \rho_{\mathrm{1p1h}}(U)\,R_n(J)$. The energy-dependent density ρ_{1p1h} is obtained from an equidistant Fermi-gas model with finite hole-depth restrictions taken into account. R_n is a Gaussian spin distribution,

$$R_n(J) = \frac{(2J+1)}{2(2\pi)^{1/2}\sigma_n^3}\exp\left[-\frac{(J+\frac{1}{2})^2}{2\sigma_n^2}\right], \quad (7)$$

with σ_n the spin cut-off parameter. The transitions are induced by an effective NN interaction which is given by a finite-range Yukawa potential with strength V_0 adjusted to reproduce the experimental (p, p') cross sections.

3   Calculational details

The QF (e,e'p) cross sections were calculated in DWIA [3], including the effect of Coulomb distortion of the electron waves, through the effective momentum approximation, which is a good approximation for light nuclei [27]. A full out-of-plane kinematics was considered, with an outgoing-proton energy up to the maximum value compatible with the energy distribution of the bound single-particle states.

The distorted waves were thus obtained from the optical potential of ref. [28] which extends up to energies of 150 MeV and the bound-state wavefunctions from a Woods-Saxon potential with a radius parameter $r_0 = 1.3$ fm, diffuseness $a = 0.6$ fm [29] and a depth fixed to reproduce the input

energy eigenvalues. The quantum numbers and energy eigenvalues of the states that can be excited were obtained from a spherical Nilsson shell model scheme [30]. Such a scheme has been adopted because it is easily extended into the continuum as required in calculating the MSD cross section. The price of consistency between bound and continuum states is, however, paid by removal energies that are somehow different from experimental values.

The energy distribution of the bound states was taken as a Lorentzian [31]

$$S(E_m) = \frac{2}{\pi} \frac{\Gamma(E_m)}{4(E_m - E_{\mathrm{F}} - E_{\mathrm{b.e.}})^2 - \Gamma^2(E_m)}, \tag{8}$$

where $E_{\mathrm{b.e.}}$ is the g.s. nucleon binding energy and E_{F} the Fermi energy of the Nilsson level scheme. The energy dependent width is given by [32]

$$\Gamma(E_m) = \frac{24(E_m - E_{\mathrm{F}})^2}{500 + (E_m - E_{\mathrm{F}})^2}, \tag{9}$$

where the energies are in MeV.

The MSD cross sections were calculated using DWUCK4 [33] to obtain the microscopic DWBA cross sections with a Yukawa effective NN potential of range 1 fm. The strength of the potential V_0 was extracted from previous studies of the systematics of the (p,p′) reaction on the atomic mass A and the incident energy [21, 22]. For sake of consistency the same distorted waves and bound-state wavefunctions were used for the calculation of the (e,e′p) and DWBA (p,p′) cross-sections.

The microscopic transitions are averaged over transferred angular momentum and residual nucleus energy by the MSD code [34] according to eq. (6) where the 1p1h state density was calculated with an average single-particle density $g = A/13$ [16] and the spin cut-off parameter of its spin distribution was given by $\sigma_2^2 = 0.28 \times 2 \times A^{2/3}$ [35]. When calculating the multistep cross sections with eq. (5) the one-step MSD cross sections are obtained at several incident energies lower than that of the excited proton of the direct (e,e′p) knockout reaction and interpolated for other values. The convolution integral in eq. (5) is then evaluated using Monte Carlo integration (MSD code [34]).

4 Results

The approach to MSD scattering of the ejectile proton presented in the previous sections was applied to the ^{40}Ca(e,e$'$p) reaction as a case example. The ^{40}Ca nucleus is an appropriate target for the statistical treatment of the MSD theory and data at high missing energy exist with the following kinematics [29]: the incident electron energy is $E_k = 497$ MeV, the electron scattering angle $\theta = 52.9°$ and the outgoing proton energy $E = 87 \pm 10$ MeV. The scattered electron energy $E_{k'}$ varied in the experiment from 350 to 410 MeV. In our calculations we fixed $E_{k'} = 350$ MeV and worked at constant $(\vec{q}, \omega)$ by varying the proton energy E accordingly. This kinematics is unable to reach missing momenta $p_m \lesssim 100$ MeV/c for the deep states, contrary to the experimental situation, where the detector acceptances also allow to probe low values of missing momenta.

In fig. 1 we show the theoretical direct (e,e$'$p) knockout and multistep cross sections as a function of the angle γ between the emitted proton $\vec{p}'$ and the momentum transfer $\vec{q}$ at four different residual nucleus energies $U_{\mathrm{res}} = E_m - E_{\mathrm{b.e.}}$. At the lowest excitation energies, the direct nucleon knockout process dominates and exhibits a strong forward-peaking. The multistep contributions are important at large scattering angles over the whole energy range. With increasing excitation energy the two-step and three-step contributions become gradually more important than the one-step direct process over most of the angular range, apart from the very small scattering angles $\gamma \leq 10°$. The domination of multistep processes at large scattering angles is expected since as a result of multistep scattering the leading proton gradually loses memory of its initial direction yielding thus increasingly symmetric angular distributions.

In order to compare with data it is useful to define the reduced cross section as

$$\rho(p_m, E_m) = \frac{1}{\sigma_{\mathrm{cc1}}} \frac{\mathrm{d}^3\sigma}{\mathrm{d}\Omega_{k'}\mathrm{d}E_{k'}\mathrm{d}\Omega}, \tag{10}$$

where σ_{cc1} is the electron-nucleon cross section taken according to ref. [36].

In fig. 2 we compare $\rho(p_m, E_m)$ integrated over two different energy ranges with the experimental data of ref. [29]. The theoretical curves are multiplied by 0.5, a factor that can be interpreted as an average spectroscopic factor.

At low missing energies the direct process dominates, whereas at high E_m the major contribution comes from two-step and three-step processes. The relative importance of multistep processes increases with the missing momentum and is analogous to the behaviour of the multistep cross sections at large scattering angles in fig. 1.

5 Conclusions

We have calculated the multi-nucleon-nucleon scattering contributions in (e,e′p) reactions in the quasi-free region using the MSD theory of ref. [15], and have shown that such processes are important at high missing energy and momentum. Therefore, one can foresee that in other kinematics involving even higher missing energy and momentum values like, e.g., in the dip region [12], multistep scattering processes would be helpful in determining the final-sate interaction.

References

[1] S. Boffi, C. Giusti, F. D. Pacati and M. Radici, *Electromagnetic Response of Atomic Nuclei* (Clarendon Press, Oxford, 1996).

[2] S. Boffi, C. Giusti, F. D. Pacati and S. Frullani, Nucl. Phys. **A319** (1979) 461.

[3] S. Boffi, C. Giusti and F. D. Pacati, Phys. Rep. **226** (1993) 1.

[4] P. E. Ulmer *et al.*, Phys. Rev. Lett. **59** (1987) 2259.

[5] L. Weinstein *et al.*, Phys. Rev. Lett. **64** (1990) 1646.

[6] I. Bobeldijk, H. P. Blok and G. van der Steenhoven, Phys. Lett. **B281** (1992) 25.

[7] I. Bobeldijk *et al.*, Phys. Rev. Lett. **73** (1994) 2684.

[8] R. W. Lourie *et al.*, Phys. Rev. Lett. **56** (1986) 2364.

[9] L. J. H. M. Kester *et al.*, Phys. Lett. **B344** (1995) 79.

[10] J. Ryckebusch *et al.*, Phys. Lett. **B333** (1994) 310.

[11] C. Ciofi degli Atti, S. Simula, L. L. Franfurt and M. I. Strikman, Phys. Rev. **C44** (1991) R7.

[12] T. Takaki, Phys. Rev. Lett. **62** (1989) 395; Phys. Rev. **39** (1989) 359.

[13] C. Giusti and F. D. Pacati, Nucl. Phys. **A571** (1994) 694.

[14] G. P. Capitani, E. De Sanctis, P. Levi-Sandri, M. Bernheim, S. Turck-Chièze, S. Frullani and J. Mougey, Nuovo Cim. **A 85** (1985) 37.

[15] H. Feshbach, A. K. Kerman and S. Koonin, Ann. Phys. (NY) **125** (1980) 429.

[16] E. Gadioli and P. E. Hodgson, *Preequilibrium Nuclear Reactions* (Clarendon Press, Oxford, 1992).

[17] M. B. Chadwick and P. G. Young, Phys. Rev. **C 47** (1993) 2255.

[18] P. Demetriou, P. Kanjanarat and P. E. Hodgson, J. Phys. G: Nucl. Part. Phys. **20** (1994) 1779.

[19] P. Demetriou, D. Phil. Thesis, University of Oxford, 1996.

[20] W. A. Richter, A. A. Cowley, R. Lindsay, J. J. Lawrie, S. V. Fortsch, J. V. Pilcher, R. Bonetti and P. E. Hodgson, Phys. Rev. **C 46** (1992) 1030.

[21] W. A. Richter, A. A. Cowley, G. C. Hillhouse, J. A. Stander, J. W. Koen, S. Y. Steyn, R. Lindsay, R. E. Julies, J. J. Lawrie, J. V. Pilcher and P. E. Hodgson, Phys. Rev. **C 49** (1994) 1001.

[22] M. B. Chadwick, P. G. Young, D. C. George and Y. Watanabe, Phys. Rev. **C 50** (1994) 996.

[23] M. B. Chadwick, S. Chiba, K. Niita, T. Maruyama, and A. Iwamoto, Phys. Rev. **C 52** (1995) 2800.

[24] A. J. Koning and J. M. Akkermans, Ann. Phys. (NY) **208** (1993) 216.

[25] R. Bonetti, J. M. Akkermans, A. J. Koning and P. E. Hodgson, Phys. Rep. **247** (1994) 1.

[26] F. C. Williams, Nucl. Phys. **A166** (1971) 231.

[27] C.Giusti and F. D. Pacati, Nucl. Phys. **A485** (1988) 461.

[28] M. M. Giannini, G. Ricco and A. Zucchiatti, Ann. Phys. (NY) **124** (1980) 208.

[29] J. Mougey, M. Bernheim, A. Bussiere, A. Gillebert, Phan Xuan Hô, M. Priou, D. Royer and G. J. Wagner, Nucl. Phys. **A262** (1976) 461.

[30] P. A. Seeger and R. C. Perisho, Los Alamos Report No. LA-3751 (1967).

[31] C. Mahaux and R. Sartor, Nucl. Phys. **A484** (1988) 275.

[32] G. E. Brown and M. Rho, Nucl. Phys. **A372** (1981) 397.

[33] P. D. Kunz and E. Rost, Computational Nuclear Physics **2**, ed. K. Langanke *et al.*(Springer Verlag, 1993), chapter 5.

[34] H. B. Olaniyi, MSD code (1992) unpublished; P. Demetriou (1995).

[35] C. Y. Fu, Nucl. Sci. Eng. **92** (1986) 440.

[36] T. de Forest, Jr., Nucl. Phys. **A392** (1983) 232.

Figure captions

Fig. 1. Differential cross section for the ^{40}Ca(e,e$'$p) reaction as a function of the angle γ between the emitted proton and the momentum transfer at four different energies U_{res} of the residual nucleus. Solid line for the direct (e,e$'$p) process; dashed, dot-dashed and dotted lines for the two-, three- and four-step processes. The total result is given by the solid line with marking dots.

Fig. 2. Reduced cross section for the ^{40}Ca(e,e$'$p) reaction as a function of the missing momentum integrated over two different missing-energy ranges. Experimental data from ref. [29], line convention for the theoretical curves as in fig. 1.